MATH 147 QUIZ 1 SOLUTIONS

1. For the function $f(x,y)$, the point $(a,b) \in \mathbb{R}^2$, and the real number $L \in \mathbb{R}$, give the epsilon-delta definition for $\lim_{(x,y)\to(a,b)} f(x,y) = L$. (2 points)

   We say that $\lim_{(x,y)\to(a,b)} f(x,y) = L$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $|(x,y)-(a,b)| < \delta$ implies that $|f(x,y) - L| < \epsilon$.

2. Determine if the limit $\lim_{(x,y)\to(0,0)} \frac{x^3+x^5}{x^2+y^2}$, exists, and if it does, find its value. (4 points)

   To solve this limit, we convert to polar coordinates. That is, make the substitutions $x = r\cos\theta$, $y = r\sin\theta$, and $r^2 = x^2 + y^2$. Upon this conversion we have

$$\lim_{(x,y)\to(0,0)} \frac{x^3+x^5}{x^2+y^2} = \lim_{r\to 0, 0\le\theta<2\pi} \frac{r^3\cos^3\theta(1+r^2\cos^2\theta)}{r^2} = \lim_{r\to 0, 0\le\theta<2\pi} r\cos^3\theta(1+r^2\cos^2\theta) = 0.$$

3. Determine if the limit $\lim_{(x,y)\to(2,1)} \frac{x-y-1}{\sqrt{x-y}-1}$ exists, and if it does, find its value. (4 points)

   We solve using the algebraic technique of *multiplying by the conjugate*. In this case, the conjugate of $\sqrt{x-y}-1$ is $\sqrt{x-y}+1$, and we multiply this on the top and bottom of the function to see:

$$\begin{aligned} \lim_{(x,y)\to(2,1)} \frac{x-y-1}{\sqrt{x-y}-1} \cdot \frac{\sqrt{x-y}+1}{\sqrt{x-y}+1} &= \lim_{(x,y)\to(2,1)} \frac{(x-y-1)(\sqrt{x-y}+1)}{x-y-1} \\ &= \lim_{(x,y)\to(2,1)} \sqrt{x-y}+1 = \sqrt{2-1}+1 = 2. \end{aligned}$$